$$E_{f_p}(\xi,\eta) = \iint A(x,y)\, e^{-i2\pi(x\xi+y\eta)}\, dx\,dy$$

$$= \int_{y=-\infty}^{\infty}\left[\int_{x=-\infty}^{\infty} A(x,y)\cos(2\pi x\xi) - i A(x,y)\sin(2\pi y\eta)\, dx\right] e^{-i2\pi y\eta}\, dy$$

$$= \int_{y=-\infty}^{\infty} P_{Re}(\xi,y)\, e^{-i2\pi y\eta}\, dy + i\int_{y=-\infty}^{\infty} P_{Im}(\xi,y)\, e^{-i2\pi y\eta}\, dy$$

where $P_{Re}(\xi,y) = \int_{x=-\infty}^{\infty} A(x,y)\cos(2\pi x\xi)\,dx$

$P_{Im}(\xi,y) = -\int_{x=-\infty}^{\infty} A(x,y)\sin(2\pi x\xi)\,dx$

$$= \int_y P_{Re}\cos(2\pi y\eta)\,dy - i\int_y P_{Re}\sin(2\pi y\eta)\,dy + i\int_y P_{Im}(\xi,y)\cos(2\pi y\eta)\,dy + i(-i)\int_y P_{Im}\sin(2\pi y\eta)\,dy$$

but for $A(x,y) = A(-x,y)$, $P_{Im}(\xi,y) = 0$ due ~~(cos integral of odd)~~ to the odd integrand.

$P_{Re}(\xi,y) = 2\int_0^{\infty} A(x,y)\cos(2\pi x\xi)\,dx$

Also, when $A(x,y)$ is real,

$$E_{f_p}(\xi,-\eta) = \int_y P_{Re}(\xi,y)\cos(2\pi y\eta)\,dy - i\int_y P_{Re}(\xi,y)\sin(2\pi y(-\eta))\,dy$$

$$= \int_y P_{Re}(\xi,y)\cos(2\pi y\eta)\,dy + i\int_y P_{Re}(\xi,y)\sin(2\pi y\eta)\,dy$$

$$= \mathrm{Re}\{E_{f_p}(\xi,\eta)\} - \mathrm{Im}\{E_{f_p}(\xi,\eta)\} = E^*(\xi,\eta) \quad \text{(A)}$$

And since $P_{Re}(\xi,y)$ is even in ξ

$$E_{f_p}(-\xi,\eta) = E_{f_p}(\xi,\eta) \quad \text{(B)} \quad \text{and} \quad E_{f_p}(-\xi,-\eta) = E^*(\xi,\eta) \quad \text{(C)}$$

Due to (A), (B), and (C), only one quadrant need be evaluated.

$$E_{fpm}(\xi,\eta) = M_{fp}(\xi,\eta)\, E_{fp}(\xi,\eta)$$ where $M_{fp}(\xi,\eta)$ is real and two-fold symmetric

$$E_{Lp}(u,v) = \int_{\xi}\int_{\eta} E_{fpm}(\xi,\eta)\, e^{-i2\pi(u\xi + v\eta)}\, d\eta\, d\xi$$

$$= \int_{\eta}\left[\int_{\xi} E_{fpm}(\xi,\eta)\, e^{-i2\pi u\xi}\, d\xi\right] e^{-i2\pi v\eta}\, d\eta$$

$$= \int_{\eta} P_{Lp}(u,\eta)\, e^{-i2\pi v\eta}\, d\eta$$ where $$P_{Lp}(u,\eta) = \int_{\xi} E_{fpm}(\xi,\eta)\, e^{-i2\pi u\xi}\, d\xi$$

Both the real and imaginary parts of $E_{fpm}(\xi,\eta)$ are even in ξ, therefore the sine integral drops out and

$$P_{Lp}(u,\eta) = 2\int_{\xi=0}^{\infty} E_{fpm}(\xi,\eta)\cos(2\pi u\xi)\, d\xi$$

$$E_{Lp}(u,v) = 2\int_{\eta=-\infty}^{\infty}\int_{\xi=0}^{\infty} E_{fpm}(\xi,\eta)\cos(2\pi u\xi)\, d\xi\, \cos(2\pi v\eta)\, d\eta$$

$$- 2i\int_{\eta=-\infty}^{\infty}\int_{\xi=0}^{\infty} E_{fpm}(\xi,\eta)\cos(2\pi u\xi)\, d\xi\, \sin(2\pi v\eta)\, d\eta$$

$$E_{Lp}(u,v) = \int_{\eta} Re\{P_{Lp}(u,\eta)\} e^{-i2\pi v\eta} d\eta + i \int_{\eta} Im\{P_{Lp}(u,\eta)\} e^{-i2\pi v\eta} d\eta$$

$$Re\{P_{Lp}(u,\eta)\} = 2 \int_{\xi=0}^{\infty} Re\{E_{fpm}(\xi,\eta)\} \cos(2\pi u\xi)\, d\xi$$

$$= Re\{P_{Lp}(u,-\eta)\} \Rightarrow (\text{even in } \eta)$$

$$Im\{P_{Lp}(u,\eta)\} = 2 \int_{\xi=0}^{\infty} Im\{E_{fpm}(\xi,\eta)\} \cos(2\pi u\xi)\, d\xi$$

$$= -Im\{P_{Lp}(u,-\eta)\} \Rightarrow (\text{odd in } \eta)$$

Now using these symmetry properties,

$$\int_{\eta} Re\{P_{Lp}(u,\eta)\} e^{-i2\pi v\eta} d\eta = 2 \cdot 2 \int_{\eta=0}^{\infty} \int_{\xi=0}^{\infty} Re\{E_{fpm}(\xi,\eta)\} \cos(2\pi u\xi)\, d\xi \cos(2\pi v\eta)\, d\eta$$

$$i \int_{\eta} Im\{P_{Lp}(u,\eta)\} e^{-i2\pi v\eta} d\eta = 2 \cdot 2\, i \int_{\eta=0}^{\infty} -i \int_{\xi=0}^{\infty} Im\{E_{fpm}(\xi,\eta)\} \cos(2\pi u\xi)\, d\xi \, \sin(2\pi v\eta)\, d\eta$$