Neil Zimmerman

Summary of Wes Traub's method for predicting RV planet detection times from the AFTA coronagraph models

This is a mathematical description inferred from Wes Traub's F77 source code, **yield.f**, now translated to **yield.py**. All constants, variables, and functions have been renamed here for clearer notation.

1. Constants

1.1. Telescope

$D_{tel} = 237$ cm	telescope diameter
$f_A = 0.826$	area factor = clear/total primary mirror area
$A_{tel} = f_A \cdot \pi (D_{tel}/2)^2 = 36439.0$ cm^2	telescope collecting area
$N_{mir} = 27$	number of reflections
$f_{mir} = 0.98$	mirror reflectivity
$\eta = {f_{mir}}^{N_{mir}} = 0.58$	telescope efficiency
$\mathcal{J}_{tel} = 0.2$ mas, 0.4 mas	jitter level (goal and baseline, respectively)

1.2. Camera

$\Delta\lambda/\lambda = 0.10$	fractional bandwidth
$Q = 0.8$	quantum efficiency
$\mu_d = 0.001$ e$^-$ pix^{-1} s^{-1}	dark mean current, per sec per pixel
$f_{EN} = \sqrt{2}$	excess noise factor
$\sigma_{rd} = 3.0$ e$^-$ pix^{-1}	s.d. of read noise, per frame per pixel
$\mu_{CIC} = 0.001$ e$^-$ pix^{-1}	mean CIC noise, per frame per pixel
$G = 1.0$	EMF gain
$\tau_{fr} = 300.0$ s	time per exposure
$\tau_{camp} = 60.0$ days	campaign duration
$M = 5$	minimum ratio between planet signal and speckle floor for detection
$N_{pix} = \pi/4 \cdot 2.5^2 = 4.9$	number of pixels in Nyquist+ sampled PSF core
$f_{pp} = 1/30,\ 1/10$	post-processing speckle suppression factor (goal, baseline)
$\lambda_0 = 0.55$ μm	reference wavelength

1.3. Target assumptions

$m_{z0} = 22.9$ mag/arcsec^2	exozodi reference surface brightness at 1 AU
$N_z = 3$	zodi enhancement factor
$\mathcal{A}_g = 0.4$	geometric albedo of planet
$e_{orb} = 0$	eccentricity (implicitly fixed at zero throughout)
$i_{orb} = 60°$	inclination angle of orbit
$\nu_{orb} = 70°$	true anomaly
$\omega_{orb} = 90°$	argument of periastron (arbitrary for a circular orbit; implicitly set to 90° by Wes)
$\mathcal{F}_0(\lambda = 0.55\ \mu\text{m}) = 10^{6.75}$ phot $\cdot$ cm$^{-2} \cdot$ s$^{-1} \cdot (\Delta\lambda/\lambda)^{-1}$	$\lambda = 0.55\ \mu$m-centered irradiance zero pt.

2. Variables read in from RV target data and coronagraph simulation result tables

$C_{speck}(\theta, \lambda, \mathcal{J}_{tel})$	speckle contrast (specific to coronagraph and assumed jitter level)
$T_{core}(\theta, \lambda)$	throughput to core of the off-axis coronagraph PSF
$T_{tot}(\theta, \lambda)$	overall throughput (transmission) of the off-axis coronagraph PSF
$\Omega_{PSF}(\theta, \lambda)$	solid angle within core of the off-axis coronagraph PSF
m_V	apparent magnitude of star
R_p	radius of planet, converted to units of AU
a_p	semi-major axis of planet's orbit in AU
$d_\star$	distance to star in pc

3. Method

The scattering phase angle, α, is fixed throughout this preliminary "science yield" exercise, but the code was written in such a way that it can be looped and repeated for arbitrary sets of orbital elements ν_{orb} and i_{orb}. Since the eccentricity, and therefore the argument of periastron, were fixed for all planets, α reduces to a function of true anomaly and inclination angle:

$$\alpha = \arccos\left(\sin\left(\nu_{orb} + \omega_{orb}\right)\sin i_{orb}\right) \tag{1}$$

$$= \arccos\left(\sin\left(\nu_{orb} + 90°\right)\sin i_{orb}\right) \tag{2}$$

$$= \arccos\left(\cos\nu_{orb}\sin i_{orb}\right) \tag{3}$$

For the $\nu = 70°$ and $i = 60°$ chosen by Wes, $\alpha = 72.8°$.

The phase angle is fed into the classical phase function for a perfectly scattering sphere, $\Phi(\alpha)$, which will scale the albedo between zero and unity depending on the star-planet-observer configuration:

$$\Phi(\alpha) = \left(\sin\alpha + (\pi - \alpha)\cos\alpha\right)/\pi, \tag{4}$$

which evaluates to $\Phi(72.8°) = 0.48$.

Angular separation between the planet and star, θ_p, is evaluated once for each planet (since α is fixed). If there are multiple (i, ν) configurations to test, this will be repeated and stored to a table.

$$\theta_p = a_p \sin(\alpha)/d_\star \tag{5}$$

The planet-to-star contrast, C_p:

$$C_p = \left(r_p/a_p\right)^2 \mathcal{A}_p \Phi(\alpha) \tag{6}$$

The above equations (1–6) are described in the chapter written by Burrows & Orton, *Giant Planet Atmospheres* for Sara Seager's *Exoplanets* textbook, in particular pages 423–424.

Irradiance of star, $\mathcal{F}_\star$

$$\mathcal{F}_\star = \mathcal{F}_0 \; 10^{-m_V/2.5} \quad [\mathrm{phot} \cdot \mathrm{cm}^{-2} \cdot \mathrm{s}^{-1} \cdot (\Delta\lambda/\lambda)^{-1}] \tag{7}$$

The PSF core photoelectron count rate of the star, $n_\star$, is evaluated as if the star's image were in the same image plane position as the target planet during the science exposure (in other words, no direct obstruction by the coronagraph, but taking into account throughput losses due to all masks):

$$n_\star = \eta \; A_{tel} \; Q \; T_{core}(\theta_p, \lambda_0) \; \Delta\lambda/\lambda \; \mathcal{F}_\star \quad [\mathrm{e}^-/\mathrm{s}] \tag{8}$$

Following from this and the planet-to-star contrast, we get the PSF core photoelectron count rate of the planet, n_p:

$$n_p = n_\star C_p \quad [\mathrm{e}^-/\mathrm{s}] \tag{9}$$

Similarly, for the speckle count rate we evaluate (by linear interpolation) the contrast curve from John Krist's simulation result table at θ_p:

$$n_{speck} = n_{\star} C_{speck}(\theta_p, \lambda_0, \mathcal{J}_{tel}) \quad [\mathrm{e}^-/\mathrm{s}] \tag{10}$$

Exozodiacal irradiance per solid angle on sky:

$$\frac{d\mathcal{F}_z}{d\Omega} = N_z \mathcal{F}_0 10^{-m_{z0}/2.5} \quad [\mathrm{phot} \cdot \mathrm{cm}^{-2} \cdot \mathrm{s}^{-1} \cdot (\Delta\lambda/\lambda)^{-1}\,\mathrm{arcsec}^{-2}] \tag{11}$$

In the version of yield.f given to us by Wes, the exozodiacal surface brightness was fixed for all planets at m_{z0}, with no dependence on stellar luminosity or star-planet separation. Since the canonical surface brightness corresponds to $3\times$ the solar system surface brightness at 1 AU separation from a sun-like star, and our inner working angles generally fall at projected separations > 1 AU from the target stars, this is a conservative assumption.

This leads to the exozodiacal photoelectron count rate, n_z:

$$n_z = \eta\ A_{tel}\ Q\ T_{tot}(\theta_p, \lambda_0)\ \Delta\lambda/\lambda\ \frac{d\mathcal{F}_z}{d\Omega}\ \Omega_{PSF}(\theta_p, \lambda_0) \quad [\mathrm{e}^-/\mathrm{s}] \tag{12}$$

He defines the background photoelectron count rate, n_{bkg} as follows:

$$n_{bkg} = f_{EN}\,(n_p + n_z + n_{speck} + \mu_d\ N_{pix} + \mu_{CIC}\ N_{pix}/\tau_{fr}) + \left(\frac{\sigma_{rd}}{G}\right)^2\ N_{pix}/\tau_{fr} \quad [\mathrm{e}^-/\mathrm{s}] \tag{13}$$

The integration time of a planet is only calculated if the signal is greater than M times the speckle noise floor after post-processing, therefore requiring $n_p > n_{p,min} := M f_{pp} n_{speck}$. This is tested for both the goal ($f_{pp} = 1/30$) and baseline ($f_{pp} = 1/10$) performance cases. If that threshold is met, then the integration τ_{int} is evaluated:

$$\tau_{int} = M^2 \frac{n_{bkg}}{n_p^2 - n_{p,min}^2} \tag{14}$$

$$= 5^2 \frac{n_{bkg}}{n_p^2 - n_{p,min}^2} \tag{15}$$

$$\tag{16}$$

The RV planet detection list for a given combination of coronagraph architecture, post-processing performance, and jitter level, is determined by sorting these integration times in ascending order, and cutting off the list at the planet where the cumulative integration time, $\sum_k \tau_{int_k}$, exceeds the campaign duration, $\tau_{camp} = 60$ days.

4. Suggested improvements

1. minor: Add dependencies of exozodiacal dust surface brightness on separation and stellar luminosity (Marc Kuchner report referred to by Wes?).

2. Supplement the conventional SNR metric with a probabilistic, matched filter treatment of detection confidence (Kasdin & Braems 2006).

3. Semi-major axis and wavelength dependence on planet albedo (Burrows models) to explore IFS characterization times with EMCCD.

4. Longer term: incorporate more realistic speckle contrast curves as they become available from lab tests and wavefront control simulations.